



Mark Scheme (Results)

Summer 2019

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F1
(WFM01/01)

Summer 2019
WFM01/01 Further Pure Mathematics F1
Mark Scheme

Question Number	Scheme		Notes	Marks	
1.	$f(x) = 5 + 4x^2 - \frac{4}{3}x^3 - \frac{7}{2x}; \ x > 0$				
(a)	$f'(x) = 8x - 4x^2 + \frac{7}{2}x^{-2}$	At least one of either $5 + 4x^2 \rightarrow \pm Ax$ or $-\frac{4}{3}x^3 \rightarrow \pm Bx^2$ or $-\frac{7}{2x} \rightarrow \pm Cx^{-2}; A, B, C \neq 0$		M1	
		Correct differentiation, which can be un-simplified or simplified		A1	
				(2)	
(b)	$f(0.5) = -\frac{7}{6}, \ f'(0.5) = 17$	Either $f(0.5) = -\frac{7}{6}$ or awrt -1.17 or truncated -1.16 or $f'(0.5) = 17$ or a correct numerical expression for either $f(0.5)$ or $f'(0.5)$ Can be implied by later working		B1	
		$\left\{ \alpha \approx 0.5 - \frac{f(0.5)}{f'(0.5)} \right\} \Rightarrow \alpha \approx 0.5 - \frac{-\frac{7}{6}}{17}$	Valid attempt at Newton-Raphson using their values of $f(0.5)$ and $f'(0.5)$	M1	
	$\left\{ \alpha = 0.56862745... \text{ or } \frac{29}{51} \right\} \Rightarrow \alpha = 0.569 \ (3 \text{ dp})$	Correct $f'(x)$ and 0.569 on first iteration (Ignore any subsequent iterations)	A1 cso cao		
	Correct differentiation followed by 0.569 (with no working seen) scores full marks in part (b)		(3)		
	(c) Way 1	$f(3) = \frac{23}{6} = 3.833333...$ $f(3.5) = -\frac{25}{6} = -4.166666...$	Attempts to evaluate both $f(3)$ and $f(3.5)$ and either $f(3) = \frac{23}{6}$ or awrt 4 or truncated 3 or $f(3.5) = -\frac{25}{6}$ or awrt -4		M1
Sign change {positive, negative} {and $f(x)$ is continuous} therefore a root $\{\beta\}$ exists in the interval $\{[3, 3.5]\}$		Both values correct awrt (or truncated) to 1sf, sign change and conclusion.		A1	
				(2)	
(d) Way 1	$\frac{\beta - 3}{\text{"3.8333..."}} = \frac{3.5 - \beta}{\text{"4.1666..."}}$ or $\frac{\beta - 3}{3.5 - \beta} = \frac{\text{"3.8333..."}}{\text{"4.1666..."}}$ or $\frac{\beta - 3}{\text{"3.8333..."}} = \frac{3.5 - \beta}{\text{"4.166..." + "3.8333..."}}$		A correct linear interpolation method. Do not allow this mark if a total of one or three negative lengths are used or if either fraction is the wrong way up. This mark may be implied.		M1
	<ul style="list-style-type: none">$\beta = \left(\frac{(3)(\text{"4.1666..."}) + (3.5)(\text{"3.8333..."})}{\text{"4.1666..." + "3.8333..."}} \right) = \left(\frac{12.5 + 13.4166...}{8} \right)$$\beta = 3 + \left(\frac{\text{"3.8333..."}}{\text{"4.1666..." + "3.8333..."}} \right)(0.5)$ or $\beta = 3 + \left(\frac{\text{"\frac{23}{6}"}}{\text{"8"}} \right)(0.5)$$\beta = 3 + \left(\frac{\text{"-3.8333..."}}{\text{"-4.1666..." + "-3.8333..."}} \right)(0.5)$		dependent on the previous M mark Rearranges to give $\beta = ...$		dM1
	$\left\{ \beta = 3.239583... \text{ or } 3\frac{23}{96} \text{ or } \frac{311}{96} \right\} \Rightarrow \beta = 3.24 \ (2 \text{ dp})$		awrt 3.24 (Ignore any subsequent iterations)		A1
					(3)
					10

Question Number	Scheme		Notes	Marks
1. (d) Way 2	$\frac{x}{\text{"3.8333..."}} = \frac{0.5 - x}{\text{"4.1666..."}}$ $x = \frac{(0.5)(\text{"3.8333..."})}{\text{"3.8333..." + "4.1666..."}} = 0.239583...$ $\Rightarrow \beta = 3 + 0.239583...$		Finds x using a correct method of similar triangles and applies "3 + their x"	M1 dM1
	$\left\{ \beta = 3.239583... \text{ or } 3\frac{23}{96} \text{ or } \frac{311}{96} \right\} \Rightarrow \beta = 3.24 \text{ (2dp)}$		awrt 3.24	A1
				(3)
1. (d) Way 3	$\frac{0.5 - x}{\text{"3.8333..."}} = \frac{x}{\text{"4.1666..."}}$ $x = \frac{(0.5)(\text{"4.1666..."})}{\text{"3.8333..." + "4.1666..."}} = 0.260416...$ $\Rightarrow \beta = 3.5 - 0.260416...$		Finds x using a correct method of similar triangles and applies "3.5 – their x"	M1 dM1
	$\left\{ \beta = 3.239583... \text{ or } 3\frac{23}{96} \text{ or } \frac{311}{96} \right\} \Rightarrow \beta = 3.24 \text{ (2dp)}$		awrt 3.24	A1
				(3)
Question 1 Notes				
1. (b)	Note	Give full marks in part (b) for correct differentiation in (a) followed by the correct answer in (b) with <u>no</u> working.		
	M1	This mark can be implied by applying at least one correct value of either $f(0.5)$ or their $f'(0.5)$ (where $f'(0.5)$ is found using their $f'(x)$) to 1 significant figure in $0.5 - \frac{f(0.5)}{f'(0.5)}$. So just writing $0.5 - \frac{f(0.5)}{f'(0.5)}$ with an incorrect ft answer on their $f'(0.5)$ scores B0M0A0.		
	Note	Give B1M1A0 for a correct $f'(x)$ in (a) followed by only $\alpha \approx 0.5 - \frac{f(0.5)}{f'(0.5)} = \frac{29}{51}$ in (b)		
	Note	Differentiating INCORRECTLY to give $f'(x) = 8x - 4x^2 + 14x^{-2}$ leads to $\alpha \approx 0.5 - \frac{-\frac{7}{6}}{59} = \frac{92}{177} = 0.5197740113... = 0.520 \text{ (3 dp)}$ This response should be given B1 M1 A0		
	Note	Differentiating INCORRECTLY to give $f'(x) = 8x - 4x^2 + 14x^{-2}$ and $\alpha \approx 0.5 - \frac{f(0.5)}{f'(0.5)} = 0.520 \text{ or truncated } 0.52 \text{ or } 0.519 \text{ or awrt } 0.520 \text{ is B1 M1 A0}$		
(c)	Note	Way 1: correct solution only Required to state both values for $f(3)$ and $f(3.5)$ correct awrt (or truncated) to 1sf along with a reason and a conclusion . Reference to change of sign or e.g. $f(3) \times f(3.5) < 0$ or $f(3) > 0 > f(3.5)$ or a diagram or < 0 and > 0 or one positive, one negative are sufficient reasons. There must be a conclusion, e.g. $\{x \text{ or } \} \beta \in [3, 3.5]$ or $\{x \text{ or } \} \beta \in (3, 3.5)$ or root lies between 3 and 3.5. Ignore the presence or absence of any reference to continuity.		
	Note	A minimal acceptable reason and conclusion is “change of sign, so $\beta \in [3, 3.5]$ ” or “change of sign, so root is between 3 and 3.5” or “change of sign, so root”		

Question 1 Notes Continued															
1. (c)	Note	<p>Way 2</p> <p>The root of $f(x) = 0$ is 3.27491258..., so they can choose x_1 which is less than 3.27491258... and choose x_2 which is greater than 3.27491258... with both x_1 and x_2 lying in the interval $[3, 3.5]$.</p> <p>M1: Finds $f(x_1)$ and $f(x_2)$ with one of these values correct awrt (or truncated) to 1sf</p> <p>A1: Both values correct awrt (or truncated) to 1sf, sign change and conclusion.</p>													
	Note	<p>Helpful Table</p> <table><tr><th>x</th><th>$f(x)$</th></tr><tr><td>3</td><td>3.83333333...</td></tr><tr><td>3.1</td><td>2.58963440...</td></tr><tr><td>3.2</td><td>1.17558333...</td></tr><tr><td>3.3</td><td>-0.41660606...</td></tr><tr><td>3.4</td><td>-2.19474509...</td></tr><tr><td>3.5</td><td>-4.16666666...</td></tr></table>	x	$f(x)$	3	3.83333333...	3.1	2.58963440...	3.2	1.17558333...	3.3	-0.41660606...	3.4	-2.19474509...	3.5
x	$f(x)$														
3	3.83333333...														
3.1	2.58963440...														
3.2	1.17558333...														
3.3	-0.41660606...														
3.4	-2.19474509...														
3.5	-4.16666666...														
1. (d)	Note	Condone writing the symbol α in place of β in part (d)													
	Note	$\frac{\beta - 3}{3.5 - \beta} = \frac{\text{"3.833..."}{\text{"-4.1666..."}}}$ is a valid method for the first M mark													
	Note	Give 1 st M1 for either $\frac{f(3)}{-f(3.5)} = \frac{\beta - 3}{3.5 - \beta}$ or $\frac{f(1.2)}{ f(1.3) } = \frac{\beta - 3}{3.5 - \beta}$ or $\frac{ f(3) }{ f(3.5) } = \frac{\beta - 3}{3.5 - \beta}$													
	Note	Give M1 dM1 A1 for the correct statement $\frac{3 f(3.5) + 3.5f(3)}{ f(3.5) + f(3)} = 3.24$													
	Note	Give M0 dM0 for $\frac{3 f(3.5) + 3.5f(3)}{ f(3.5) + f(3)} = \frac{3\text{"-4.166..."} + 3.5\text{"3.8333..."}{(\text{"-4.166..."} + \text{"3.8333..."})}$													
	Note	Give M1 dM1 for the correct statement $\beta = \frac{3.5 + 3k}{k + 1}$, where k is defined as $k = \frac{ f(3.5) }{f(3)} = \frac{4.1666...}{3.8333...} = 1.086957...$													
	Note	Give M1 dM1 for the correct statement $\beta = \frac{3 + 3.5c}{c + 1}$, where c is defined as $c = \frac{f(3)}{ f(3.5) } = \frac{3.8333...}{4.1666...} = 0.92$													
	Note	$\frac{\beta - 3}{3.5 - \beta} = \frac{\text{"3.8333..."}{\text{"4.1666..."}}} \Rightarrow \beta = 3.24$ with no intermediate working is M1 dM1 A1													
	Note	$\frac{\beta - 3}{3.8333...} = \frac{3.5 - \beta}{-4.1666...} \Rightarrow \beta = -2.75$ is M0 dM0 A0													
	Note	$\frac{\beta - 3}{-3.8333...} = \frac{3.5 - \beta}{-4.1666...} \Rightarrow \beta = 3.24$ is M1 dM1 A1													
Note	$\frac{\beta - 3}{3.5 - \beta} = \frac{\text{"4.1666..."}{\text{"3.8333..."} } \Rightarrow \beta = 3.260416... is M0 dM0 A0$														

Question Number	Scheme	Notes	Marks
1. (d) Way 4	<ul style="list-style-type: none"> $y - \frac{23}{6} = \frac{-\frac{25}{6} - \frac{23}{6}}{3.5 - 3}(x - 3) \Rightarrow 0 - \frac{23}{6} = \frac{-\frac{25}{6} - \frac{23}{6}}{3.5 - 3}(x - 3)$ $y - -\frac{25}{6} = \frac{-\frac{25}{6} - \frac{23}{6}}{3.5 - 3}(x - 3.5) \Rightarrow 0 - -\frac{25}{6} = \frac{-\frac{25}{6} - \frac{23}{6}}{3.5 - 3}(x - 3.5)$ 	Complete method of finding a line joining the points (3, f(3)), (3.5, f(3.5)) followed by setting $y = 0$	M1
	$\Rightarrow x = \dots$ or $\beta = \dots$	dependent on the previous M mark Rearranges to give $x = \dots$ or $\beta = \dots$	dM1
	$\left\{ x \text{ or } \beta = 3.239583\dots \text{ or } 3\frac{23}{96} \text{ or } \frac{311}{96} \right\} \Rightarrow \beta = 3.24 \text{ (2dp)}$	awrt 3.24	A1
			(3)

Question Number	Scheme		Notes	Marks
2.	$\mathbf{M} = \begin{pmatrix} k-12 & 3 \\ 4 & k \end{pmatrix}$, where k is a real constant			
	$\left\{ \det(\mathbf{M}) = (k-12)k - 4(3) \text{ and area ratio} = \frac{320}{20} = 16 \right\}$			
	$20(k(k-12) - 4(3)) = 320$ or $20(k(k-12) - 4(3)) = -320$ or $k(k-12) - 4(3) = \frac{320}{20}$ or $k(k-12) - 4(3) = -\frac{320}{20}$		20(applied $\det(\mathbf{M})$) = ± 320 , o.e. Note: Allow $320(\text{applied } \det(\mathbf{M})) = \pm 20$, o.e.	M1
			At least one correct equation in k that can be simplified or un-simplified	A1
	$k^2 - 12k - 28 = 0$, $k^2 - 12k + 4 = 0$ { $20k^2 - 240k - 560 = 0$, $20k^2 - 240k + 80 = 0$ }			
	$(k-14)(k+2) = 0$ or $(k-6)^2 - 36 + 4 = 0$ to give $k = \dots$	dependent on the previous M mark At least one correct method (e.g. factorising, applying the quadratic formula, completing the square or calculator) of solving a 3TQ to give $k = \dots$		dM1
	$k = 14, -2, 6 + 4\sqrt{2}, 6 - 4\sqrt{2}$		At least two of either $k = 14, k = -2$, $k = 6 + 4\sqrt{2}$ or $k = 6 - 4\sqrt{2}$	A1
			All four correct values of k	A1
				(5)
				5
	Question 2 Notes			
2.	Note	Allow 1 st M1 for any of <ul style="list-style-type: none"> $320(k(k-12) - 4(3)) = 20$ $320(k(k-12) - 4(3)) = -20$ $k(k-12) - 4(3) = \frac{20}{320}$ $k(k-12) - 4(3) = -\frac{20}{320}$ which can be simplified or un-simplified.		
	Note	Allow 1 st M1 for any of <ul style="list-style-type: none"> $20(k(k-12) + 4(3)) = 320$ $320(k(k-12) + 4(3)) = 20$ $20(k(k-12) + 4(3)) = -320$ $320(k(k-12) + 4(3)) = -20$ or equivalent, which can be simplified or un-simplified.		
	Note	Give 1 st M0 for any of <ul style="list-style-type: none"> $(k(k-12) - 4(3)) = (20)(320)$ $(k(k-12) + 4(3)) = (20)(320)$ 		
	Note	Give dM1 for using a calculator to write down at least one correct root for their 3TQ		
	Note	For the 1 st A1 mark <ul style="list-style-type: none"> condone truncated 11.6 or awrt 11.7 in place of $k = 6 + 4\sqrt{2}$ condone awrt 0.34 in place of $k = 6 - 4\sqrt{2}$ 		
	Note	Allow $k = 6 + \sqrt{32}$ instead of $k = 6 + 4\sqrt{2}$ and/or $k = 6 - \sqrt{32}$ instead of $k = 6 - 4\sqrt{2}$ for any of the final two accuracy marks.		
	Note	Allow final A1 (isw) for $k = 14, -2, 6 + 4\sqrt{2}, 6 - 4\sqrt{2}$, awrt 11.6, $6 - 4\sqrt{2}$, awrt 0.34		
	Note	Give 2 nd A0 (i.e. the penultimate mark) for finding only one correct value for k as a result of rejecting (or ignoring) correct values for k		
	Note	Give final A0 if any of $k = 14, -2, 6 + 4\sqrt{2}, 6 - 4\sqrt{2}$ are rejected		
	Note	Give final A0 for extra solutions in addition to $k = 14, -2, 6 + 4\sqrt{2}, 6 - 4\sqrt{2}$		

Question 2 Notes Continued		
2.	Note	$320(k(k-12)-4(3))=20$ leads to $16k^2-192k-193=0$ and $k=12.9327\dots, -0.9327\dots$ $320(k(k-12)-4(3))=-20$ leads to $16k^2-192k-191=0$ and $k=12.9236\dots, -0.9236\dots$
	Note	$20(k(k-12)+4(3))=320$ leads to $k^2-12k-4=0$ and $k=12.3245\dots, -0.3245\dots$ $20(k(k-12)+4(3))=-320$ leads to $k^2-12k+28=0$ and $k=8.8284\dots, 3.1715\dots$
	Note	$320(k(k-12)+4(3))=20$ leads to $16k^2-192k+191=0$ and $k=10.9053\dots, 1.0946\dots$ $320(k(k-12)+4(3))=-20$ leads to $16k^2-192k+193=0$ and $k=10.8925\dots, 1.1074\dots$

Question Number	Scheme	Notes	Marks
3.	(i) $z^* - 3z = \frac{5i}{3-i}$; (ii) $w = -4 + 5i$, (b) $\arg(w+k) = \frac{\pi}{2}$, (c) $ w+ci = 4\sqrt{5}$		
(i) Way 1	$\{z^* - 3z = \} (a-ib) - 3(a+ib)$	Left hand side = $(a-ib) - 3(a+ib)$ Can be implied by e.g. $-2a - 4bi$ Note: Can be seen (or implied) anywhere in their solution	B1
	$\dots\dots\dots = \frac{5i}{(3-i)(3+i)}$	Multiplies numerator and denominator of the right-hand side by $3+i$ or $-3-i$	M1
	$\dots\dots\dots = \frac{15i-5}{10}$	Applies $i^2 = -1$ to give right-hand side = $\frac{15i-5}{10}$ or equivalent	A1
	So, $-2a - 4bi = -\frac{1}{2} + \frac{3}{2}i$ $\Rightarrow a = \frac{1}{4}, b = -\frac{3}{8} \Rightarrow z = \frac{1}{4} - \frac{3}{8}i$	dependent on the previous B and M marks Equates either real parts or imaginary parts to give at least one of $a = \dots$ or $b = \dots$	ddM1
		$z = \frac{1}{4} - \frac{3}{8}i$ or $z = 0.25 - 0.375i$ or $z = \frac{1}{4} + \left(-\frac{3}{8}i\right)$	A1
			(5)
(i) Way 2	$\{z^* - 3z = \} (a-ib) - 3(a+ib)$	Left hand side = $(a-ib) - 3(a+ib)$ Can be implied by e.g. $-2a - 4bi$ Note: Can be seen (or implied) anywhere in their solution	B1
	$(-2a - 4bi)(3-i) = \dots\dots\dots$	Multiplies their $(-2a - 4bi)$ by $(3-i)$	M1
	$-6a + 2ai - 12bi - 4b = \dots\dots\dots$	Applies $i^2 = -1$ to give left-hand side = $-6a + 2ai - 12bi - 4b$ or equivalent	A1
	So, $(-6a - 4b) + (2a - 12b)i = 5i$ gives $-6a - 4b = 0$, $2a - 12b = 5$ $\Rightarrow a = \frac{1}{4}, b = -\frac{3}{8} \Rightarrow z = \frac{1}{4} - \frac{3}{8}i$	dependent on the previous B and M marks Equates both real parts and imaginary parts and solves simultaneously to give at least one of $a = \dots$ or $b = \dots$	ddM1
		$z = \frac{1}{4} - \frac{3}{8}i$ or $z = 0.25 - 0.375i$ or $z = \frac{60}{240} - \frac{15}{40}i$	A1
			(5)
(ii)(a)	e.g. $\arg w = \pi - \tan^{-1}\left(\frac{5}{4}\right)$ or $= \frac{\pi}{2} + \tan^{-1}\left(\frac{4}{5}\right)$ or $= -\pi - \tan^{-1}\left(\frac{5}{4}\right)$	Uses trigonometry to find an expression for $\arg w$ so that $\arg w$ is in the range $(1.58\dots, 3.14\dots)$ or $(90^\circ, 180^\circ)$ or $(-4.71\dots, -3.15\dots)$ or $(-270^\circ, -180^\circ)$	M1
	$\arg w = \pi - 0.896055\dots = 2.245537\dots \{= 2.25 \text{ (2 dp)}\}$ or $\arg w = -\pi - 0.896055\dots = -4.037648\dots \{= -4.04 \text{ (2 dp)}\}$	awrt 2.25 or awrt -4.04 or awrt 8.53 or awrt -10.32	A1
	{ Note: $\arg w = 128.6598\dots^\circ$ or $-231.3401\dots^\circ$ is M1 A0}		(2)
(b)	$\{\arg(-4+5i+k) = \frac{\pi}{2} \Rightarrow -4+k=0 \Rightarrow\} k=4$	$k=4$	B1
			(1)
(c)	$ -4+5i+ci = 4\sqrt{5}$ $\Rightarrow -4+(5+c)i = 4\sqrt{5}$ $\Rightarrow (-4)^2 + (5+c)^2 = (4\sqrt{5})^2$	Squares and adds the real and imaginary parts of $w+ci$ and sets equal to either $(4\sqrt{5})^2$ or $4\sqrt{5}$ $(-4)^2 + (5+c)^2 = (4\sqrt{5})^2$ o.e. Allow the equivalent result $\sqrt{(-4)^2 + (5+c)^2} = 4\sqrt{5}$	M1
	$16 + (5+c)^2 = 80 \Rightarrow (5+c)^2 = 64 \Rightarrow c = \dots$ or $16 + (5+c)^2 = 80 \Rightarrow c^2 + 10c - 39 = 0$ $\Rightarrow (c+13)(c-3) = 0 \Rightarrow c = \dots$	dependent on the previous M mark Solves their quadratic in c to give $c = \dots$	dM1
	$c = -13, 3$	$c = -13, 3$	A1
			(4)
			12

Question 3 Notes		
3. (i)	Note	Allow alternative ways of defining z . E.g. $z = x + iy$ and $z^* = x - iy$ with $x \equiv a$ and $y \equiv b$
	Note	Give final A0 for defining $z = a + ib$, finding $a = \frac{1}{4}$, $b = -\frac{3}{8}$ but not stating $z = \frac{1}{4} - \frac{3}{8}i$
	Note	<u>Alternative:</u> Some may define $z = x - iy$ and $z^* = x + iy$ This gives $\{z^* - 3z = \} (x + iy) - 3(x - iy) = -2x + 4yi$ So, $-2x + 4yi = -\frac{1}{2} + \frac{3}{2}i \Rightarrow x = \frac{1}{4}, y = \frac{3}{8} \Rightarrow z = \frac{1}{4} - \frac{3}{8}i$
(ii) (a)	Note	Allow M1 (implied) for awrt 2.2, awrt -3.8 , truncated -4.0 , awrt 129° , truncated 128° or awrt -231°
(ii) (c)	Note	$ -4 + (5 + c)i = 4\sqrt{5} \Rightarrow (-4)^2 - (5 + c)^2 = (4\sqrt{5})^2$ unless recovered is 1 st M0
	Note	$ -4 + (5 + c)i = 4\sqrt{5} \Rightarrow -16 + (5 + c)^2 = (4\sqrt{5})^2$ unless recovered is 1 st M0
	Note	$ -4 + 5i + ci = 4\sqrt{5} \Rightarrow (-4)^2 + (5)^2 + c^2 = (4\sqrt{5})^2$ unless recovered is 1 st M0
	Note	If a 3TQ is formed in c then a correct method (e.g. factorising, applying the quadratic formula, completing the square or calculator) of solving a 3TQ is required to give $c = \dots$
	Note	Give dM1 for using a calculator to write down at least one correct root for their 3TQ
	Note	Having achieved a correct $16 + 25 + 10c + c^2 = 80$ give final dM1 A1 marks for writing down $c = -13, 3$ from no working.
	Note	Give final A0 for either <ul style="list-style-type: none"> $c = -13, 3 \Rightarrow c = 3$ $c = -13, 3 \Rightarrow c = -13$ $c = 3, c = -13$ (reject) $c = 3$ (reject), $c = -13$

Question Number	Scheme	Notes	Marks
4. (a) Way 1	$\sum_{r=1}^{3k} (4r+1) = 4 \cdot \frac{1}{2} (3k)(3k+1) + 3k$	Either $\sum_{r=1}^{3k} 4r \rightarrow 4 \cdot \frac{1}{2} (3k)(3k+1)$ or $\sum_{r=1}^{3k} 1 \rightarrow 3k$	M1
		Correct expression, simplified or un-simplified	A1
	$= 6k(3k+1) + 3k = 18k^2 + 9k$		
	$= 9k(2k+1) \quad \{p=9\}$	Obtains $9k(2k+1)$ with no errors	A1 cso
			(3)
(a) Way 2	$\sum_{r=1}^k (4r+1) = 4 \cdot \frac{1}{2} (k)(k+1) + k$	Both $\sum_{r=1}^k 4r \rightarrow 4 \cdot \frac{1}{2} (k)(k+1)$ and $\sum_{r=1}^k 1 \rightarrow k$	M1
	$= 2k(k+1) + k = 2k^2 + 3k$		
	$\sum_{r=1}^{3k} (4r+1) = 2(3k)(3k+1) + 3k = 2(3k)^2 + 3(3k)$	Correct expression, simplified or un-simplified	A1
	$= 18k^2 + 9k$		
	$= 9k(2k+1) \quad \{p=9\}$	Obtains $9k(2k+1)$ with no errors	A1 cso
			(3)
(b) Way 1	$\sum_{r=1}^k 2r^2 = \sum_{r=1}^{3k} (4r+1)$		
	$2 \cdot \frac{1}{6} k(k+1)(2k+1) = 9k(2k+1)$	Sets $\lambda k(k+1)(2k+1)$ equal to "9" $k(2k+1)$ or their answer from part (a), $\lambda \neq 0$, to give an equation in k only	M1
	$\frac{1}{3} (k+1) = 9 \Rightarrow k = 26$	dependent on the previous M mark Cancels out two terms or factorises out two terms and solves a linear equation in k to give $k = \dots$	dM1
		$k = 26$ only	A1
			(3)
(b) Way 2	$2 \cdot \frac{1}{6} k(k+1)(2k+1) = 9k(2k+1)$	Sets $\lambda k(k+1)(2k+1)$ equal to "9" $k(2k+1)$ or their answer from part (a), $\lambda \neq 0$, to give an equation in k only	M1
	$2k^3 + 3k^2 + k = 54k^2 + 27k$ $2k^3 - 51k^2 - 26k = 0$ $k(2k^2 - 51k - 26) = 0$ $(2k+1)(k-26) = 0 \Rightarrow k = 26$	dependent on the previous M mark Cancels out or factorises k and a correct method (e.g. factorising, applying the quadratic formula, completing the square or calculator) of solving a 3TQ to give $k = \dots$	dM1
		$k = 26$ only	A1
			(3)
(b) Way 3	$2 \cdot \frac{1}{6} k(k+1)(2k+1) = 9k(2k+1)$	Sets $\lambda k(k+1)(2k+1)$ equal to "9" $k(2k+1)$ or their answer from part (a), $\lambda \neq 0$, to give an equation in k only	M1
	$k(k+1)(2k+1) = 27k(2k+1)$ $k(k+1) - 27k = 0 \Rightarrow k^2 - 26k = 0$ $k(k-26) \Rightarrow k = 26$	dependent on the previous M mark Cancels out two terms or factorises out two terms and solves a linear equation in k to give $k = \dots$	dM1
		$k = 26$ only	A1
			(3)
			6

Question 4 Notes		
4. (a)	Note	Give M1A1 for $\sum_{r=1}^{3n} (4r+1) = 4 \cdot \frac{1}{2} (3n)(3n+1) + 3n$
	Note	Give M1A1A0 for $\sum_{r=1}^{3n} (4r+1) = 4 \cdot \frac{1}{2} (3n)(3n+1) + 3n = 18n^2 + 9n = 9n(2n+1)$ without reference to $\sum_{r=1}^{3k} (4r+1) = 9k(2k+1)$
	Note	Give M1A1A1 for $\sum_{r=1}^{3n} (4r+1) = 4 \cdot \frac{1}{2} (3n)(3n+1) + 3n = 18n^2 + 9n = 9n(2n+1) \Rightarrow \sum_{r=1}^{3k} (4r+1) = 9k(2k+1)$
	Note	Way 2: Give M1 for $\sum_{r=1}^n (4r+1) = 4 \cdot \frac{1}{2} (n)(n+1) + n$
	Note	Give final A0 for cancelling down their final answer $9k(2k+1)$ in part (a) E.g. $\sum_{r=1}^{3k} (4r+1) = 4 \cdot \frac{1}{2} (3k)(3k+1) + 3k = 18k^2 + 9k = 9k(2k+1) = k(2k+1)$ gets M1 A1 A0
	Note	Give M0 A0 A0 for writing e.g. $k=1 \Rightarrow \sum_1^{3(1)} (4r+1) = p(1)((2(1)+1)) \Rightarrow 5+9+13 = 3p \Rightarrow p=9$ with no evidence of applying $\sum_{r=1}^{3k} 4r \rightarrow 4 \cdot \frac{1}{2} (3k)(3k+1)$ or $\sum_{r=1}^{3k} 1 \rightarrow 3k$
	Note	You can give M1 1 st A1 marks in part (a) for work recovered for $\sum_{r=1}^{3k} (4r+1) = 4 \cdot \frac{1}{2} (3k)(3k+1) + 3k$ in part (b)
(b)	Note	Condone giving 1 st M1 for setting $\lambda k(k+1)(2k+1)$ equal to "9" $k(k+1)$ {slip}
	Note	Give A0 for giving more than one value of k as their final answer.
	Note	Where applicable, for A1, <ul style="list-style-type: none"> $k=0$ and/or $k=-\frac{1}{2}$ needs to be rejected leaving $k=26$ as their final answer. $k=26$ needs to be indicated as their final answer.
	Note	Way 2: Using fractions gives <ul style="list-style-type: none"> $\frac{2}{3}k^3 + k^2 + \frac{1}{3}k = 18k^2 + 9k \Rightarrow \frac{2}{3}k^3 - 17k^2 - \frac{26}{3}k = 0 \Rightarrow \frac{2}{3}k^2 - 17k - \frac{26}{3} = 0$ $\Rightarrow k = \frac{17 \pm \sqrt{(-17)^2 - 4(\frac{2}{3})(-\frac{26}{3})}}{2(\frac{2}{3})} = \frac{17 \pm \sqrt{\frac{2809}{9}}}{\frac{4}{3}} = \frac{17 \pm \frac{53}{3}}{\frac{4}{3}} \Rightarrow k = 26$
	Note	Way 3: E.g. Give dM0 for $k^2 + k - 27k = 0$ leading directly to $k = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-27)}}{2(1)}$

Question Number	Scheme	Notes	Marks
5.	$\mathbf{A} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2\sqrt{3} & -7 \\ -4 & 5\sqrt{3} \end{pmatrix}$		
(a)	Rotation	Rotation or rotate (condone turn)	B1
	60 degrees {anti-clockwise}	60 degrees or $\frac{\pi}{3}$ or 300 degrees clockwise or $\frac{5\pi}{3}$ clockwise	B1 o.e.
	about (0, 0)	This mark is dependent on at least one of the previous B marks being given. about (0, 0) or about <i>O</i> or about the origin	dB1
	Note: Give 2 nd B0 for 60 degrees clockwise o.e.		(3)
(b)	$\{\mathbf{A}^6 = \} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	Correct matrix	B1
			(1)
(c) Way 1	$\mathbf{B}^{-1} = \frac{1}{2} \begin{pmatrix} 5\sqrt{3} & 7 \\ 4 & 2\sqrt{3} \end{pmatrix}$	Correct matrix for \mathbf{B}^{-1} , which can be simplified or un-simplified	B1
	$\{\mathbf{C} = \mathbf{B}^{-1}\mathbf{A}\} = \frac{1}{2} \begin{pmatrix} 5\sqrt{3} & 7 \\ 4 & 2\sqrt{3} \end{pmatrix} \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \dots$	Applies (their \mathbf{B}^{-1}) \mathbf{A} , where (their \mathbf{B}^{-1}) $\neq \mathbf{B}$, and finds at least one element (or at least one element calculation) of their matrix \mathbf{C} Note: Allow one slip in copying down \mathbf{A}	M1
	$= \frac{1}{2} \begin{pmatrix} 6\sqrt{3} & -4 \\ 5 & -\sqrt{3} \end{pmatrix}$ or $= \begin{pmatrix} 3\sqrt{3} & -2 \\ \frac{5}{2} & -\frac{1}{2}\sqrt{3} \end{pmatrix}$	dependent on the previous B1M1 marks At least 2 elements in \mathbf{C} are correct	A1
		All elements in \mathbf{C} are correct	A1
			(4)
(c) Way 2	$\{\mathbf{BC} = \mathbf{A} \Rightarrow\}$ $\begin{pmatrix} 2\sqrt{3} & -7 \\ -4 & 5\sqrt{3} \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	Correct statement using 2×2 matrices. All 3 matrices must contain four elements. Can be implied by the 4 correct equations that are below.	B1
	$2\sqrt{3}a - 7c = \frac{1}{2}, 2\sqrt{3}b - 7d = -\frac{\sqrt{3}}{2}$ $-4a + 5\sqrt{3}c = \frac{\sqrt{3}}{2}, -4b + 5\sqrt{3}d = \frac{1}{2}$ and finds at least one of either a, b, c or d	Applies $\mathbf{BC} = \mathbf{A}$ and attempts to solve simultaneous equations in a and c or b and d and finds at least one of either a, b, c or d	M1
	$= \frac{1}{2} \begin{pmatrix} 6\sqrt{3} & -4 \\ 5 & -\sqrt{3} \end{pmatrix}$ or $= \begin{pmatrix} 3\sqrt{3} & -2 \\ \frac{5}{2} & -\frac{1}{2}\sqrt{3} \end{pmatrix}$	dependent on the previous B1M1 marks At least 2 elements in \mathbf{C} are correct	A1
	or $a = 3\sqrt{3}, b = -2, c = \frac{5}{2}, d = -\frac{1}{2}\sqrt{3}$	All elements in \mathbf{C} are correct	A1
			(4)
			8

Question 5 Notes		
5. (a)	Note	Writing “60 degrees” by itself implies by convention “60 degrees anti-clockwise”. So, <ul style="list-style-type: none"> • “Rotation 60 degrees about O” is B1 B1 B1 • “Rotation 60 degrees clockwise about O” is B1 B0 B1
	Note	Writing down “60 degrees anti-clockwise about O ” with no reference to “rotation” or “turn” is B0 B1 B1
	Note	“original point” is not acceptable in place of the word “origin”.
	Note	Give B0 B0 B0 for a combination of 2 or more transformations.
(b)	Note	Give B0 for writing down \mathbf{I} without reference to $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
	Note	Allow B1 for writing down \mathbf{I}_2 without reference to $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
(c)	Note	Allow B1 for $\frac{1}{(2\sqrt{3})(5\sqrt{3}) - (-7)(-4)} \begin{pmatrix} 5\sqrt{3} & 7 \\ 4 & 2\sqrt{3} \end{pmatrix}$ or $\frac{1}{30-28} \begin{pmatrix} 5\sqrt{3} & 7 \\ 4 & 2\sqrt{3} \end{pmatrix}$
	Note	Allow B1 for $\begin{pmatrix} 5\sqrt{3} & 7 \\ 4 & 2\sqrt{3} \end{pmatrix} \frac{1}{(2\sqrt{3})(5\sqrt{3}) - (-7)(-4)}$ or $\begin{pmatrix} 5\sqrt{3} & 7 \\ 4 & 2\sqrt{3} \end{pmatrix} \frac{1}{30-28}$
	Note	You can ignore previous working prior to their finding $\mathbf{B}^{-1}\mathbf{A}$ (i.e. you can ignore an incorrect statement such as $\mathbf{A} = \mathbf{CB}$)

Question Number	Scheme		Notes	Marks
6.	$2x^2 + x + 4 = 0$ has roots α, β			
(a)	$\alpha + \beta = -\frac{1}{2}, \alpha\beta = 2$		Both $\alpha + \beta = -\frac{1}{2}$ and $\alpha\beta = 2$	B1
				(1)
(b)(i)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \dots$		Use of a correct identity for $\alpha^2 + \beta^2$ May be implied by their work	M1
	$= \left(-\frac{1}{2}\right)^2 - 2(2) = -\frac{15}{4}$		$-\frac{15}{4}$ or -3.75 or $-3\frac{3}{4}$ from correct working	A1 cso
	(ii)	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = \dots$ or $= (\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta) = \dots$ or $= (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta) = \dots$	Use of a correct identity for $\alpha^3 + \beta^3$ May be implied by their work	M1
		$= \left(-\frac{1}{2}\right)^3 - 3(2)\left(-\frac{1}{2}\right) = \frac{23}{8}$ or $= \left(-\frac{1}{2}\right)\left(\left(-\frac{1}{2}\right)^2 - 3(2)\right) = \frac{23}{8}$ or $= \left(-\frac{1}{2}\right)\left(-\frac{15}{4} - 2\right) = \frac{23}{8}$	$\frac{23}{8}$ or 2.875 or $2\frac{7}{8}$ from correct working	A1 cso
(c)	$\Sigma = \alpha^3 + \frac{1}{\beta} + \beta^3 + \frac{1}{\alpha}$ $= \alpha^3 + \beta^3 + \frac{\alpha + \beta}{\alpha\beta}$	$\Sigma = \frac{\alpha^3\beta + 1}{\beta} + \frac{\alpha\beta^3 + 1}{\alpha}$ $= \frac{\alpha\beta(\alpha^3 + \beta^3) + (\alpha + \beta)}{\alpha\beta}$	Simplifies $\frac{1}{\beta} + \frac{1}{\alpha}$ to give $\frac{\alpha + \beta}{\alpha\beta}$ (can be implied) and uses at least two of their $\alpha^3 + \beta^3, \alpha + \beta$ or $\alpha\beta$ in an attempt to find a numerical value for the sum of $\left(\alpha^3 + \frac{1}{\beta}\right)$ and $\left(\beta^3 + \frac{1}{\alpha}\right)$	M1
	e.g. $= \frac{23}{8} + \frac{\left(-\frac{1}{2}\right)}{2} = \frac{21}{8}$ or $= \frac{2\left(\frac{23}{8}\right) + \left(-\frac{1}{2}\right)}{2}$			
	$\Pi = \left(\alpha^3 + \frac{1}{\beta}\right)\left(\beta^3 + \frac{1}{\alpha}\right)$ $= (\alpha\beta)^3 + \alpha^2 + \beta^2 + \frac{1}{\alpha\beta}$	$\Pi = \left(\frac{\alpha^3\beta + 1}{\beta}\right)\left(\frac{\alpha\beta^3 + 1}{\alpha}\right)$ $= \frac{\alpha^4\beta^4 + \alpha^3\beta + \alpha\beta^3 + 1}{\alpha\beta}$ $= \frac{(\alpha\beta)^4 + \alpha\beta(\alpha^2 + \beta^2) + 1}{\alpha\beta}$	Expands $\left(\alpha^3 + \frac{1}{\beta}\right)\left(\beta^3 + \frac{1}{\alpha}\right)$ to give 4 terms and uses at least one of their $\alpha\beta$ or $\alpha^2 + \beta^2$ in an attempt to find a numerical value for the product	M1
	e.g. $= (2)^3 + \left(-\frac{15}{4}\right) + \frac{1}{2} = \frac{19}{4}$ or $= \frac{(2)^4 + 2\left(-\frac{15}{4}\right) + 1}{2}$			
	$x^2 - \frac{21}{8}x + \frac{19}{4} = 0$		Applies $x^2 - (\text{sum})x + \text{product}$ (can be implied), for their numerical values of the sum and product. Note: " $=0$ " is not required for this mark	M1
	$8x^2 - 21x + 38 = 0$		Any integer multiple of $8x^2 - 21x + 38 = 0$, including the " $=0$ "	A1 cso
				(4)
				9

Question 6 Notes		
6. (b)(i)	Note	Writing a correct $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ without attempting to substitute at least one of either their $\alpha + \beta$ or their $\alpha\beta$ into $(\alpha + \beta)^2 - 2\alpha\beta$ is M0
	Note	An incorrect $\alpha + \beta = \frac{1}{2}$, $\alpha\beta = 2$ from (a) leading to $\alpha^2 + \beta^2 = \left(\frac{1}{2}\right)^2 - 2(2) = -\frac{15}{4}$ is M1 A0
	Note	Give M1 A1 for writing down $\alpha^2 + \beta^2 = -\frac{15}{4}$, if they give $\alpha + \beta = -\frac{1}{2}$, $\alpha\beta = 2$ in (a)
(b)(ii)	Note	Allow M1 A1 for $\alpha^3 + \beta^3 = (\alpha^2 + \beta^2)(\alpha + \beta) - \alpha\beta(\alpha + \beta) = \left(-\frac{15}{4}\right)\left(-\frac{1}{2}\right) - (2)\left(-\frac{1}{2}\right) = \frac{23}{8}$
	Note	E.g. writing a correct $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ without attempting to substitute at least one of either their $\alpha + \beta$ or their $\alpha\beta$ into $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ is M0
	Note	E.g. writing a correct $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$ without attempting to substitute at least one of either their $\alpha + \beta$, their $\alpha^2 + \beta^2$ or their $\alpha\beta$ into $(\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$ is M0
	Note	Give M1 A1 for writing down $\alpha^3 + \beta^3 = \frac{23}{8}$, if they give $\alpha + \beta = -\frac{1}{2}$, $\alpha\beta = 2$ in (a)
(b)	ALT	<p>They can use the equation $2x^2 + x + 4 = 0$ with roots α, β to give</p> $\begin{cases} 2\alpha^2 + \alpha + 4 = 0 \\ 2\beta^2 + \beta + 4 = 0 \end{cases} \Rightarrow 2\alpha^2 + 2\beta^2 + \alpha + \beta + 8 = 0$ <p>So, $\alpha^2 + \beta^2 = \frac{1}{2}(-(\alpha + \beta) - 8) = \frac{1}{2}\left(-\left(-\frac{1}{2}\right) - 8\right) = \frac{1}{2}\left(\frac{1}{2} - 8\right) = -\frac{15}{4}$</p> $\begin{cases} 2\alpha^3 + \alpha^2 + 4\alpha = 0 \\ 2\beta^3 + \beta^2 + 4\beta = 0 \end{cases} \Rightarrow 2\alpha^3 + 2\beta^3 + \alpha^2 + \beta^2 + 4\alpha + 4\beta = 0$ <p>So, $\alpha^3 + \beta^3 = \frac{1}{2}(-(\alpha^2 + \beta^2) - 4(\alpha + \beta)) = \frac{1}{2}\left(-\left(-\frac{15}{4}\right) - 4\left(-\frac{1}{2}\right)\right) = \frac{1}{2}\left(\frac{15}{4} + 2\right) = \frac{23}{8}$</p>
(a)	Note	Give B0 for $\alpha, \beta = \frac{-1 + \sqrt{31}i}{4}, \frac{-1 - \sqrt{31}i}{4}$ and then stating that $\alpha + \beta = -\frac{1}{2}$, $\alpha\beta = 2$
	Note	Give B0 for $\alpha + \beta = \frac{-1 + \sqrt{31}i}{4} + \frac{-1 - \sqrt{31}i}{4} = -\frac{1}{2}$ and $\alpha\beta = \left(\frac{-1 + \sqrt{31}i}{4}\right)\left(\frac{-1 - \sqrt{31}i}{4}\right) = 2$
(b)(i)	Note	Give M0 A0 for $\alpha^2 + \beta^2 = \left(\frac{-1 + \sqrt{31}i}{4}\right)^2 + \left(\frac{-1 - \sqrt{31}i}{4}\right)^2 = -\frac{15}{4}$
(b)(ii)	Note	Give M0 A0 for $\alpha^3 + \beta^3 = \left(\frac{-1 + \sqrt{31}i}{4}\right)^3 + \left(\frac{-1 - \sqrt{31}i}{4}\right)^3 = \frac{23}{8}$
(b)	Note	<p>Using $\frac{-1 + \sqrt{31}i}{4}, \frac{-1 - \sqrt{31}i}{4}$ to find $\alpha + \beta = -\frac{1}{2}$, $\alpha\beta = 2$ followed by</p> <ul style="list-style-type: none"> $\alpha^2 + \beta^2 = \left(-\frac{1}{2}\right)^2 - 2(2) = -\frac{15}{4}$, scores M1 A0 in (b)(i) e.g. $\alpha^3 + \beta^3 = \left(-\frac{1}{2}\right)^3 - 3(2)\left(-\frac{1}{2}\right) = \frac{23}{8}$, scores M1 A1 in (b)(ii)
(c)	Note	A correct method leading to $p = 8, q = -21, r = 38$ without writing a final answer of $8x^2 - 21x + 38 = 0$ is final M1 A0

Question 6 Notes Continued		
6. (c)	Note	<p>Using $\frac{-1+\sqrt{31}i}{4}, \frac{-1-\sqrt{31}i}{4}$ explicitly to find the sum and product of $\alpha^3 + \frac{1}{\beta}$ and $\beta^3 + \frac{1}{\alpha}$</p> <ul style="list-style-type: none"> i.e. sum = $\left(\frac{-1+\sqrt{31}i}{4}\right)^3 + \frac{1}{\left(\frac{-1-\sqrt{31}i}{4}\right)} + \left(\frac{-1-\sqrt{31}i}{4}\right)^3 + \frac{1}{\left(\frac{-1+\sqrt{31}i}{4}\right)} = \frac{21}{8}$ ie. product = $\left(\left(\frac{-1+\sqrt{31}i}{4}\right)^3 + \frac{1}{\left(\frac{-1-\sqrt{31}i}{4}\right)}\right)\left(\left(\frac{-1-\sqrt{31}i}{4}\right)^3 + \frac{1}{\left(\frac{-1+\sqrt{31}i}{4}\right)}\right) = \frac{19}{4}$ $x^2 - \frac{21}{8}x + \frac{19}{4} = 0 \Rightarrow 8x^2 - 21x + 38 = 0$ <p>scores M0 M0 M1 A0 in part (c).</p>
	Note	<p>Using $\frac{-1+\sqrt{31}i}{4}, \frac{-1-\sqrt{31}i}{4}$ to find $\alpha + \beta = -\frac{1}{2}, \alpha\beta = 2$</p> <p>and applying $\alpha + \beta = -\frac{1}{2}, \alpha\beta = 2$ can potentially score full marks in (c). E.g.</p> <ul style="list-style-type: none"> sum = $\alpha^3 + \beta^3 + \frac{\alpha + \beta}{\alpha\beta} = \frac{23}{8} + \frac{(-\frac{1}{2})}{2} = \frac{21}{8}$ product = $(\alpha\beta)^3 + \alpha^2 + \beta^2 + \frac{1}{\alpha\beta} = (2)^3 + \left(-\frac{15}{4}\right) + \frac{1}{2} = \frac{19}{4}$ $x^2 - \frac{21}{8}x + \frac{19}{4} = 0 \Rightarrow 8x^2 - 21x + 38 = 0$
	Note	Give final M0 for $\sum = \frac{21}{8}, \Pi = \frac{19}{4}$ leading to $x^2 - \frac{21}{8}x + \frac{19}{4} = 0$ (without recovery)
	Note	Allow final M1 for $\sum = \frac{21}{8}, \Pi = \frac{19}{4}$ with $x^2 - (\text{sum})x + (\text{product})$ leading to $x^2 - \frac{21}{8}x + \frac{19}{4} = 0$
	Note	An alternative method uses a correct $\left(x - \alpha^3 - \frac{1}{\beta}\right)\left(x - \beta^3 - \frac{1}{\alpha}\right) = 0$
	Note	Allow 1 st M1 and/or 2 nd M1 for using an incorrect $\left(x - \alpha^3 + \frac{1}{\beta}\right)\left(x - \beta^3 + \frac{1}{\alpha}\right) = 0$
	Note	Give final M0 for an incorrect $\left(x - \alpha^3 + \frac{1}{\beta}\right)\left(x - \beta^3 + \frac{1}{\alpha}\right) = 0$ unless recovered
	Note	<p>When expanding $\left(\alpha^3 + \frac{1}{\beta}\right)\left(\beta^3 + \frac{1}{\alpha}\right)$ to give $(\alpha\beta)^3 + \alpha^2 + \beta^2 + \frac{1}{\alpha\beta}$, some will write $\frac{\alpha + \beta}{\alpha\beta}$ in place of $\frac{1}{\alpha\beta}$</p> <p>So, allow 2nd M1 for expanding $\left(\alpha^3 + \frac{1}{\beta}\right)\left(\beta^3 + \frac{1}{\alpha}\right)$ to give $(\alpha\beta)^3 + \alpha^2 + \beta^2 + \frac{\alpha + \beta}{\alpha\beta}$ and using at least one of their $\alpha\beta$ or $\alpha^2 + \beta^2$ in an attempt to find a numerical value for the product.</p>

Question Number	Scheme		Notes	Marks
7.	$f(z) = z^4 - 6z^3 + az^2 - 44z + b$; a, b are real constants. $z = -1 - 3i$ is given.			
(a)	$-1 + 3i$		$-1 + 3i$	B1
				(1)
(b)	$z^2 + 2z + 10$	Attempt to expand $(z \pm (-1 - 3i))(z \pm (-1 + 3i))$ or any valid method to establish a quadratic factor e.g. $z = -1 \pm 3i \Rightarrow z + 1 = \pm 3i \Rightarrow z^2 + 2z + 1 = -9$ or sum of roots $= -2$, product of roots $= 10$ to give $z^2 \pm (\text{their sum})z \pm (\text{their product})$		M1
		$z^2 + 2z + 10$		A1
	$\{f(z) = \} (z^2 + 2z + 10)(z^2 - 8z + 18)$	Attempts to find the other quadratic factor e.g. using long division to obtain $z^2 + kz + \dots, k = \text{value} \neq 0$ e.g. factorising/equating coefficients to obtain $f(z) = (z^2 + 2z + 10)(z^2 \pm kz \pm c)$, $k = \text{value} \neq 0, c$ can be 0		M1
		$z^2 - 8z + 18$ seen in their working		A1
	$\{z^2 - 8z + 18 = 0 \Rightarrow \}$			
	<ul style="list-style-type: none"> $z = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(18)}}{2(1)}$ $(z - 4)^2 - 16 + 18 = 0 \Rightarrow z = \dots$ 	dependent on only the previous M mark Correct method of applying the quadratic formula or completing the square for solving their 3TQ on their 2 nd quadratic factor		dM1
	$\{z = \} 4 \pm \sqrt{2}i$	$4 + \sqrt{2}i$ and $4 - \sqrt{2}i$		A1
				(6)
				7
Question 7 Notes				
7. (a)	Note	Give B1 for either $4 + \sqrt{2}i$ or $4 - \sqrt{2}i$		
(b)	Note	The values of the constants, i.e. $a = 12, b = 180$ do not have to be found explicitly.		
	Note	You can assume $x \equiv z$ for solutions in this part		
	Note	Give final dM1A1 for $z^2 - 8z + 18 = 0 \Rightarrow z = 4 + \sqrt{2}i, 4 - \sqrt{2}i$ with no intermediate working.		
	Note	They must be solving a 3TQ " $Az^2 + Bz + C$ " where A, B, C are all numerical values $\neq 0$ for the final dM1 mark.		
	Note	Special Case: If their 2nd quadratic factor $z^2 + Bz + C$ can be factorised then give Special Case 3 rd dM1 for correct factorisation leading to $z = \dots$ Otherwise, give 3 rd dM0 for applying a method of factorisation to solve their 3TQ.		
	Note	Reminder: Method mark for solving a 3TQ Formula: $Az^2 + Bz + C = 0 \Rightarrow$ Attempt to use the correct formula (with values for A, B, C) Completing the Square: $z^2 + Bz + C = 0 \Rightarrow \left(z \pm \frac{B}{2}\right)^2 \pm q \pm C = 0, q \neq 0$, leading to $z = \dots$		
	Note:	Comparing coefficients: $f(z) = (z^2 + 2z + 10)(z^2 + \alpha z + \beta) \equiv z^4 - 6z^3 + az^2 - 44z + b$ $z^3: \alpha + 2 = -6 \Rightarrow \alpha = -8; z: 2\beta + 10\alpha = -44 \Rightarrow 2\beta - 80 = -44 \Rightarrow \beta = 18$ yielding 2 nd quadratic factor $= z^2 - 8z + 18$ Also, constant: $10\beta = b \Rightarrow b = 180; z^2: \beta + 2\alpha + 10 = a \Rightarrow a = 18 - 16 + 10 = 12$		

Question 7 Notes Continued		
7. (b)	Note:	<p><u>Long division:</u></p> $ \begin{array}{r} z^2 - 8z + 18 \\ z^2 + 2z + 10 \overline{) z^4 - 6z^3 + az^2 - 44z + b} \\ \underline{z^4 + 2z^3 + 10z^2} \\ -8z^3 + (a - 10)z^2 - 44z \\ \underline{-8z^3 - 16z^2 - 80z} \\ (a + 6)z^2 + 36z + b \\ \underline{18z^2 + 36z + 180} \\ 0 \end{array} $ <p>Also, note $a = 12, b = 180$</p>
	Note	Ignore errors in long division for the 2 nd A1 mark and/or the 3 rd A1 mark.
	Note	Ignore errors in stating $a = 12, b = 180$ for the 2 nd A1 mark and/or the 3 rd A1 mark.
	Note	The solutions $4 \pm \sqrt{2}i$ need to follow on from a correct $z^2 - 8z + 18$ in order to gain the final A mark.
	Note	Give final A0 for writing $\frac{8 \pm 2\sqrt{2}i}{2}$ followed by either $4 \pm 2\sqrt{2}i$ or $8 \pm \sqrt{2}i$

Question Number	Scheme	Notes	Marks	
8.	$f(n) = 3^{4n-2} + 2^{6n-3}$ is divisible by 17			
Way 1	$f(1) = 3^2 + 2^3 = 17$ {is divisible by 17}	$f(1) = 17$ is the minimum	B1	
	$f(k+1) - f(k) = 3^{4(k+1)-2} + 2^{6(k+1)-3} - (3^{4k-2} + 2^{6k-3})$	Attempts $f(k+1) - f(k)$	M1	
	$f(k+1) - f(k) = 80(3^{4k-2}) + 63(2^{6k-3})$			
	$= 80(3^{4k-2} + 2^{6k-3}) - 17(2^{6k-3})$ or $= 63(3^{4k-2} + 2^{6k-3}) + 17(3^{4k-2})$	$80(3^{4k-2} + 2^{6k-3})$ or $80f(k)$; $-17(2^{6k-3})$ $63(3^{4k-2} + 2^{6k-3})$ or $63f(k)$; $+17(3^{4k-2})$	A1;	A1
	$f(k+1) = 80(3^{4k-2} + 2^{6k-3}) - 17(2^{6k-3}) + f(k)$ or $f(k+1) = 63(3^{4k-2} + 2^{6k-3}) + 17(3^{4k-2}) + f(k)$ or $f(k+1) = 80f(k) - 17(2^{6k-3}) + f(k)$ or $f(k+1) = 63f(k) + 17(3^{4k-2}) + f(k)$	dependent on at least one of the previous A marks being gained Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ and/or $(3^{4k-2} + 2^{6k-3})$	dM1	
	If the result is <u>true for $n = k$</u> , then it is <u>true for $n = k + 1$</u> . As the result has been shown to be <u>true for $n = 1$</u> , then the result is true for all $n \in \mathbb{Z}^+$		A1 cso	
			(6)	
Way 2	$f(1) = 3^2 + 2^3 = 17$ {is divisible by 17}	$f(1) = 17$ is the minimum	B1	
	$f(k+1) = 3^{4(k+1)-2} + 2^{6(k+1)-3}$	Attempts $f(k+1)$	M1	
	$f(k+1) = 81(3^{4k-2}) + 64(2^{6k-3})$			
	$= 81(3^{4k-2} + 2^{6k-3}) - 17(2^{6k-3})$ or $= 64(3^{4k-2} + 2^{6k-3}) + 17(3^{4k-2})$	$81(3^{4k-2} + 2^{6k-3})$ or $81f(k)$; $-17(2^{6k-3})$ $64(3^{4k-2} + 2^{6k-3})$ or $64f(k)$; $+17(3^{4k-2})$	A1;	A1
	$f(k+1) = 81(3^{4k-2} + 2^{6k-3}) - 17(2^{6k-3})$ or $f(k+1) = 64(3^{4k-2} + 2^{6k-3}) + 17(3^{4k-2})$ or $f(k+1) = 81f(k) - 17(2^{6k-3})$ or $f(k+1) = 64f(k) + 17(3^{4k-2})$	dependent on at least one of the previous A marks being gained Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ and/or $(3^{4k-2} + 2^{6k-3})$	dM1	
	If the result is <u>true for $n = k$</u> , then it is <u>true for $n = k + 1$</u> . As the result has been shown to be <u>true for $n = 1$</u> , then the result is true for all $n \in \mathbb{Z}^+$		A1 cso	
			(6)	
Way 3	General Method: Using $f(k+1) - mf(k)$, $m \in \mathbb{Z}$			
	$f(1) = 3^2 + 2^3 = 17$ {is divisible by 17}	$f(1) = 17$ is the minimum	B1	
	$f(k+1) - mf(k) = 3^{4(k+1)-2} + 2^{6(k+1)-3} - m(3^{4k-2} + 2^{6k-3})$	Attempts $f(k+1) - mf(k)$	M1	
	$f(k+1) - mf(k) = (81-m)(3^{4k-2}) + (64-m)(2^{6k-3})$			
	$= (81-m)(3^{4k-2} + 2^{6k-3}) - 17(2^{6k-3})$ or $= (64-m)(3^{4k-2} + 2^{6k-3}) + 17(3^{4k-2})$	$(81-m)(3^{4k-2} + 2^{6k-3})$ or $(81-m)f(k)$; $-17(2^{6k-3})$ $(64-m)(3^{4k-2} + 2^{6k-3})$ or $(64-m)f(k)$; $+17(3^{4k-2})$	A1;	A1
	$f(k+1) = (81-m)(3^{4k-2} + 2^{6k-3}) - 17(2^{6k-3}) + mf(k)$ or $f(k+1) = (64-m)(3^{4k-2} + 2^{6k-3}) + 17(3^{4k-2}) + mf(k)$ or $f(k+1) = (81-m)f(k) - 17(2^{6k-3}) + mf(k)$ or $f(k+1) = (64-m)f(k) + 17(3^{4k-2}) + mf(k)$	dependent on at least one of the previous A marks being gained Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ and/or $(3^{4k-2} + 2^{6k-3})$	dM1	
	If the result is <u>true for $n = k$</u> , then it is <u>true for $n = k + 1$</u> . As the result has been shown to be <u>true for $n = 1$</u> , then the result is true for all $n \in \mathbb{Z}^+$		A1 cso	
			(6)	
			6	

Question Number	Scheme	Notes	Marks
8.	$f(n) = 3^{4n-2} + 2^{6n-3}$ is divisible by 17		
Way 4	General Method: Using $f(k+1) - mf(k)$, $m \in \mathbb{Z}$		
	$f(1) = 3^2 + 2^3 = 17$ {is divisible by 17}	$f(1) = 17$ is the minimum	B1
	$f(k+1) - mf(k) = 3^{4(k+1)-2} + 2^{6(k+1)-3} - m(3^{4k-2} + 2^{6k-3})$	Attempts $f(k+1) - mf(k)$	M1
	$f(k+1) - mf(k) = (81-m)(3^{4k-2}) + (64-m)(2^{6k-3})$		
	E.g. $m = 47 \Rightarrow f(k+1) - 47f(k) = 34(3^{4k-2}) + 17(2^{6k-3})$	$m = 47$ and $34(3^{4k-2})$	A1
		$m = 47$ and $17(2^{6k-3})$	A1
	$f(k+1) = 34(3^{4k-2}) + 17(2^{6k-3}) + 47f(k)$	dependent on at least one of the previous A marks being gained Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$	dM1
	If the result is <u>true for $n = k$</u> , then it is <u>true for $n = k + 1$</u> . As the result has been shown to be <u>true for $n = 1$</u> , then the result <u>is true for all n</u> ($\in \mathbb{Z}^+$)		A1 cso
		(6)	
	In Way 4 there are many alternatives. See below for examples of alternatives where $m = 30$ and $m = 13$		
	The A1A1dM1 marks for some alternatives using $f(k+1) - mf(k)$, $m \in \mathbb{Z}$		
Way 4.1	$f(k+1) = 81(3^{4k-2}) + 64(2^{6k-3})$		
	$= 30(3^{4k-2}) + 30(2^{6k-3}) + 51(3^{4k-2}) + 34(2^{6k-3})$		
	$= 30(3^{4k-2} + 2^{6k-3}) + 51(3^{4k-2}) + 34(2^{6k-3})$	$m = 30$ and $51(3^{4k-2})$	A1
		$m = 30$ and $34(2^{6k-3})$	A1
	$f(k+1) = 30(3^{4k-2} + 2^{6k-3}) + 51(3^{4k-2}) + 34(2^{6k-3})$ or $f(k+1) = 30f(k) + 51(3^{4k-2}) + 34(2^{6k-3})$	dependent on at least one of the previous A marks being gained Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ and/or $(3^{4k-2} + 2^{6k-3})$	dM1
Way 4.2	$f(k+1) = 81(3^{4k-2}) + 64(2^{6k-3})$		
	$= 13(3^{4k-2}) + 13(2^{6k-3}) + 68(3^{4k-2}) + 51(2^{6k-3})$		
	$= 13(3^{4k-2} + 2^{6k-3}) + 68(3^{4k-2}) + 51(2^{6k-3})$	$m = 13$ and $68(3^{4k-2})$	A1
		$m = 13$ and $51(2^{6k-3})$	A1
	$f(k+1) = 13(3^{4k-2} + 2^{6k-3}) + 68(3^{4k-2}) + 51(2^{6k-3})$ or $f(k+1) = 13f(k) + 68(3^{4k-2}) + 51(2^{6k-3})$	dependent on at least one of the previous A marks being gained Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ and/or $(3^{4k-2} + 2^{6k-3})$	dM1

Question 8 Notes							
	Note	$f(n) = 3^{4n-2} + 2^{6n-3}$ can be written as $f(n) = 3^{4n-2} + 8^{2n-1}$					
Way 5	$f(n) = 3^{4n-2} + 2^{6n-3} = 3^{4n-2} + 8^{2n-1}$						
	$f(1) = 3^2 + 8^1 = 17$ {is divisible by 17}			$f(1) = 17$ is the minimum		B1	
	$f(k+1) - f(k) = 3^{4(k+1)-2} + 8^{2(k+1)-1} - (3^{4k-2} + 8^{2k-1})$			Attempts $f(k+1) - f(k)$		M1	
	$f(k+1) - f(k) = 80(3^{4k-2}) + 63(8^{2k-1})$						
	$= 80(3^{4k-2} + 8^{2k-1}) - 17(8^{2k-1})$ or $= 63(3^{4k-2} + 8^{2k-1}) + 17(3^{4k-2})$	$80(3^{4k-2} + 8^{2k-1})$ or $80f(k)$;		$-17(8^{2k-1})$		A1;	A1
		$63(3^{4k-2} + 8^{2k-1})$ or $63f(k)$;		$+17(3^{4k-2})$			
	$f(k+1) = 80(3^{4k-2} + 8^{2k-1}) - 17(8^{2k-1}) + f(k)$ or $f(k+1) = 63(3^{4k-2} + 8^{2k-1}) + 17(3^{4k-2}) + f(k)$ or $f(k+1) = 80f(k) - 17(8^{2k-1}) + f(k)$ or $f(k+1) = 63f(k) + 17(3^{4k-2}) + f(k)$			dependent on at least one of the previous A marks being gained Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ and/or $(3^{4k-2} + 8^{2k-1})$			dM1
	If the result is <u>true for $n = k$</u> , then it is <u>true for $n = k + 1$</u> . As the result has been shown to be <u>true for $n = 1$</u> , then the result <u>is true for all $n \in \mathbb{Z}^+$</u>						A1 cso
						(6)	
	Note	Some students may set $f(k) = 17M$ and so may prove the following general results <ul style="list-style-type: none">$\{f(k+1) = 81f(k) - 17(2^{6k-3})\} \Rightarrow f(k+1) = 1377M - 17(2^{6k-3})$ or $= 17(3^4M - 2^{6k-3})$$\{f(k+1) = 64f(k) + 17(3^{4k-2})\} \Rightarrow f(k+1) = 1088M + 17(3^{4k-2})$ or $= 17(2^6M + 3^{4k-2})$					
	Note	Final A1 mark is dependent on all previous marks being scored in Q8					
	Note	Final A1: There must be a correct final expression for $f(k+1)$ and a correct conclusion. The conclusion must convey the ideas of all four underlined points either at the end of their solution or as a narrative in their solution.					
	Note	Allow as part of their conclusion “true for all positive values of n ”					
	Note	Allow as part of their conclusion “true for all values of n ”					
	Note	Allow as part of their conclusion “true for all $n \in \mathbb{N}$ ”					
	Note	Referring to n as a real number in their conclusion (e.g. true for all $n \in \mathbb{R}$) is final A0					
	Note	Condone $n \in \mathbb{Z}^*$ as part of their conclusion for the final A1 mark					
	Note	Allow $f(k+1) = 3^4f(k) - 17(2^{6k-3})$ as a correct alternative to $f(k+1) = 81f(k) - 17(2^{6k-3})$					
	Note	Allow $f(k+1) = 2^6f(k) + 17(3^{4k-2})$ as a correct alternative to $f(k+1) = 64f(k) + 17(3^{4k-2})$					

Question Number	Scheme		Notes	Marks
9.	$C: y^2 = 4ax$; $P(ap^2, 2ap)$ lies on C ; circle: $(x - 10a)^2 + y^2 = \frac{9}{4}a^2$			
(a)	$y = 2\sqrt{a}x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \sqrt{a}x^{-\frac{1}{2}} = \frac{\sqrt{a}}{\sqrt{x}}$		$\frac{dy}{dx} = \pm kx^{-\frac{1}{2}}; k \neq 0$	M1
	$2y \frac{dy}{dx} = 4a$		$ky \frac{dy}{dx} = c; k, c \neq 0$	
	$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = 2a \left(\frac{1}{2ap} \right)$		their $\frac{dy}{dt} \times \frac{1}{\text{their } \frac{dx}{dt}}$; Condone $t \equiv p$	
	{At P , $x = ap^2$, $y = 2ap \Rightarrow$ } $\frac{dy}{dx} = \frac{1}{p}$		Correct calculus work leading to $m_T = \frac{1}{p}$	A1
	So, at P , $m_N = -p$	Applies $m_N = \frac{-1}{m_T}$, to find m_N in terms of p , where m_T is found using calculus. Can be implied by later working.		M1
	either $y - 2ap = -p(x - ap^2)$	or $y - 0 = -p(x - 10a)$	Correct straight line method for an equation of a normal, where $m_N (\neq m_T)$ is found using calculus.	M1
	$0 - 2ap = -p(10a - ap^2)$ $\Rightarrow p = \dots \Rightarrow x = \dots$ or $y = \dots$	$2ap - 0 = -p(ap^2 - 10a)$	dependent on the previous M mark Complete method to find either the x or y coordinate of P	dM1
	or $2ap - 0 = -p(x - 10a) \Rightarrow x = \dots$			
	either $x = 8a$, $y = 4\sqrt{2}a$ or $P(8a, 4\sqrt{2}a)$		Either $x = 8a$ or $y = 4\sqrt{2}a$ or $y = \text{awrt } 5.66a$	A1
			$P(8a, 4\sqrt{2}a)$ or both $x = 8a$ and $y = 4\sqrt{2}a$	A1
Note: $p = 2\sqrt{2}$ or $\sqrt{8}$. Note: Ignore the additional solution $P(8a, -4\sqrt{2}a)$				(7)
(b) Way 1	Area $SBP = \frac{1}{2}(10a - a)(4\sqrt{2}a)$		$\frac{1}{2}(10a - a)(\text{their } y_P \text{ from (a)})$	M1
	$= 18\sqrt{2}a^2$		$18\sqrt{2}a^2$	A1
				(2)
(c) Way 1	$PB = \sqrt{(10a - 8a)^2 + (4\sqrt{2}a)^2} \{= 6a\}$		Complete Pythagoras method for finding length PB	M1
	$PR = 6a - 1.5a$		dependent on the previous M mark $PR = \text{"their } 6a" - 1.5a$	dM1
	$PR = 4.5a$		$PR = 4.5a$	A1
				(3)
(c) Way 2	$p = 2\sqrt{2} \Rightarrow l: y = -2\sqrt{2}x + 20\sqrt{2}a$ $(x - 10a)^2 + (-2\sqrt{2}x + 20\sqrt{2}a)^2 = \frac{9}{4}a^2$ $\Rightarrow 36x^2 - 720ax + 3591a^2 = 0$ $\Rightarrow 9(2x - 21a)(2x - 19a) = 0 \Rightarrow x = \dots$ $\Rightarrow R(9.5a, \sqrt{2}a)$		Substitutes their equation of l into the circle equation followed by a correct method for solving their 3TQ to give $x = \dots$ or $y = \dots$	M1
	$PR = \sqrt{(9.5a - 8a)^2 + (\sqrt{2}a - 4\sqrt{2}a)^2}$		dependent on the previous M mark Complete applied Pythagoras method for finding the distance between their P and their R	dM1
	$PR = 4.5a$		$PR = 4.5a$	A1
				(3)

Question Number	Scheme		Notes	Marks
9. (b) Way 2	$\text{Area } SBP = \frac{1}{2} \begin{vmatrix} a & 10a & "8a" & a \\ 0 & 0 & "4\sqrt{2}a" & 0 \end{vmatrix}$ $= \frac{1}{2} 0 - 0 + 40\sqrt{2}a^2 - 0 + 0 - 4\sqrt{2}a^2 $		Complete applied method for finding area SBP using $S(a, 0)$, $B(10a, 0)$ and their P from (a)	M1
	$= 18\sqrt{2}a^2$			A1
				(2)
9. (c) Way 3	$x_R = 10a - 1.5 \cos \left(\tan^{-1} \left(\frac{"4\sqrt{2}a"}{10a - "8a"} \right) \right)$ $y_R = 1.5 \sin \left(\tan^{-1} \left(\frac{"4\sqrt{2}a"}{10a - "8a"} \right) \right)$		Uses their P from (a) in a correct method for writing down either x_R or y_R	M1
	$\Rightarrow R(9.5a, \sqrt{2}a)$			
	$PR = \sqrt{(9.5a - 8a)^2 + (\sqrt{2}a - 4\sqrt{2}a)^2}$		dependent on the previous M mark Complete applied Pythagoras method for finding the distance between their P and their R	dM1
	$PR = 4.5a$			
			$PR = 4.5a$	A1
			(3)	
Question 9 Notes				
9. (a)	Note	Allow 1 st M1 1 st A1 (sufficient use of calculus) for $\{m_T = \frac{4a}{2y}\}$ which leads to $\{m_T = \frac{1}{p}\}$		
	Note	Allow 1 st M1 1 st A1 (sufficient use of calculus) for $\{m_T = \frac{\sqrt{a}}{x}\}$ which leads to $\{m_T = \frac{1}{p}\}$		
	Note	Give 3 rd M1 for either <ul style="list-style-type: none">$2ap = "(-p)"(ap^2) + c \Rightarrow y = "(-p)"x + \text{their } c$ or$0 = "(-p)"(10a) + c \Rightarrow y = "(-p)"x + \text{their } c$		
	Note	Writing coordinates the wrong way around E.g. finding $x = 8a$, $y = 4\sqrt{2}a$ followed by $(4\sqrt{2}a, 8a)$ is final A0		
	Note	Give final A0 for $(8a, 5.65685...a)$ without reference to $y = 4\sqrt{2}a$ or $2\sqrt{8}a$		
	Note	Accept $y_P = 2\sqrt{8}a$ written in place of $y_P = 4\sqrt{2}a$ for the final A1 A1 marks		
	Note	Special Case If they write down either $\frac{dy}{dx} = \frac{1}{p}$, $m_T = \frac{1}{p}$ or $m_N = -p$ with no evidence of using calculus then they can gain any of or all the final 4 marks in part (a).		
	ALT	Alternative Method for the 3rd M mark and 4th M mark		
	$\{B(10a, 0), P(ap^2, 2ap) \Rightarrow \}$ $m_{BP} = \frac{2ap - 0}{ap^2 - 10a} = -p$	Finds gradient of BP and sets the result equal to the gradient of their normal		3 rd M1
	$\Rightarrow p = \dots \Rightarrow x = \dots$ or $y = \dots$	dependent on the previous M mark Complete method to find either the x or y coordinate of P		4 th M1

Question 9 Notes Continued		
9. (b)	Note	Give A0 25.4558... a^2 without reference to $18\sqrt{2}a^2$
	Note	Condone one slip of either writing 9 for $10a - a$ or writing " $4\sqrt{2}$ " instead of " $4\sqrt{2}a$ " for the M mark in (b)
(c)	Note	<p>Way 2: For reference,</p> $(x - 10a)^2 + (-2\sqrt{2}x + 20\sqrt{2}a)^2 = \frac{9}{4}a^2$ $x^2 - 20ax + 100a^2 + 8x^2 - 160ax + 800a^2 = \frac{9}{4}a^2$ $9x^2 - 180ax + 900a^2 = \frac{9}{4}a^2$ $9x^2 - 180ax + \frac{3591}{4}a^2 = 0 \quad \text{or} \quad 9x^2 - 180ax + 897.75a^2 = 0$ <p>or $x^2 - 20ax + 99.75a^2 = 0$ or $4x^2 - 80ax + 399a^2 = 0$</p> $x = \frac{180a \pm \sqrt{(180a)^2 - 4(9)(\frac{3591}{4})a^2}}{2(9)} = \frac{180a \pm 9a}{2(9)}$ $x = \frac{189a}{18}, \frac{171a}{18} = 10.5a, 9.5a$
	Note	<p>The method $PB = \sqrt{(10a - "8a")^2 + ("4\sqrt{2}a")^2}$ needs to be referred to in part (c) or the result of $PB = \sqrt{(10a - "8a")^2 + ("4\sqrt{2}a")^2}$ needs to be used in part (c) to gain the M mark in part (c)</p>